Varianta 38

III.

13. a) Numărul maxim de buchete este 6.

b) $P(E)=\dfrac{nr.\ cazuri\ \ favorabile}{nr.\ cazuri\ \ posibile}=\dfrac{3}{6}=\dfrac{1}{2}$.

14. a) $E(x)=\dfrac{x+1}{x^2+1}:\dfrac{x+3-4}{4(x-1)}\cdot\left(1-\dfrac{1}{x+1}\right)=\dfrac{x+1}{x^2+1}\cdot\dfrac{4(x-1)}{x-1}\cdot\dfrac{x}{x+1}=\dfrac{4x}{x^2+1}$.

b) $\left(x^2+1\right)\cdot E(x)\le 1\Rightarrow 4x\le 1\Rightarrow x\le\dfrac{1}{4},\ x\in\mathbf{R}-\{-1;1\}\Rightarrow x\in(-\infty;-1)\cup\left(-1;\dfrac{1}{4}\right]$.

c) $E(a)=\dfrac{4a}{a^2+1}\in\mathbf{Z}$, $a\in\mathbf{Z}-\{-1;1\}$. $\left(a^2+1\right)\mid\left(4a^2\right)$ și $\left(a^2+1\right)\mid\left(4a^2+4\right)\Rightarrow\left(a^2+1\right)\mid 4$

Rezultă: $\left(a^2+1\right)\in\{1,\ 2,\ 4\}\Rightarrow a^2\in\{0;\ 1;\ 3\}\Rightarrow a\in\left\{0;-1;1;-\sqrt{3};\sqrt{3}\right\}$, $a\in\mathbf{Z}-\{-1;\ \ 1\}\Rightarrow a=0$.

15. b) $AM=MN=9$ cm, deci $\triangle MAN$ este isoscel.

c) $V_{VABC}=\dfrac{4\sqrt{6}\cdot 36\sqrt{3}}{3}=144\sqrt{2}\ \text{cm}^3$.

d) $A_{\Delta MAN}=\dfrac{PN\cdot AM}{2}=\dfrac{MN\cdot AN\cdot\sin(M\hat{N}A)}{2}\Rightarrow 6\sqrt{2}\cdot 9=9\cdot 6\sqrt{3}\cdot\sin(M\hat{N}A)\Rightarrow$

$\sin(M\hat{N}A)=\dfrac{\sqrt{2}}{\sqrt{3}}=\dfrac{\sqrt{6}}{3}$.